High-order asymptotics for wave scattering in random multi-scale media: from medical ultrasound imaging to scattering by thin layers

Laure Giovangigli*†1

[1]Unité de Mathématiques Appliquées, ENSTA (UMA-ENSTA) – École Nationale Supérieure de Techniques Avancées – 828, boulevard des Maréchaux 91762 Palaiseau Cedex, France

Abstract

We study in this talk the scattering in random media with micro-structures of size *epsilon* small compared to the wavelength of the incident wave. We show in two different examples how a high-order asymptotic expansion of the solution with respect to *epsilon* can be derived and we quantify the error between the exact solution and those approximate models using quantitative homogenization techniques. We also illustrate the convergence results with numerical simulations.

The first example focuses on quantitative ultrasound imaging and the underlying mathematical model. In soft tissues, the measured echoes come from numerous weakly contrasted unresolved scatterers. We aim at providing a mathematical framework for wave propagation in tissue-mimicking random multi-scale media. We derive a quantitative asymptotic expansion of the field measured in ultrasound imaging with respect to the size of the scatterers. This approximate model accounts for the potential effective anisotropy of the medium and depends on the local random distribution of the scatterers in the medium, making path for the inverse problem.

Secondly we study the time-harmonic scattering by a heterogeneous object covered with a thin layer of randomly distributed nanoparticles. The size of the particles, their distance between each other and the layer thickness are all of order *epsilon.* Solving numerically Maxwell's equations in this context is very costly. To circumvent this, we propose via a multi-scale asymptotic expansion of the solution, an effective model where the layer of particles and the object are replaced by an equivalent boundary condition. The coefficients in this equivalent boundary condition depend on the solutions to corrector problems of Laplace type defined on unbounded random domains. We prove that those problems are well-posed under the assumption that the particles are distributed given a stationary mixing random point process.

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*Speaker

†Corresponding author: laure.giovangigli@ensta.fr